A Number System with Golden Base

Matt DeLong

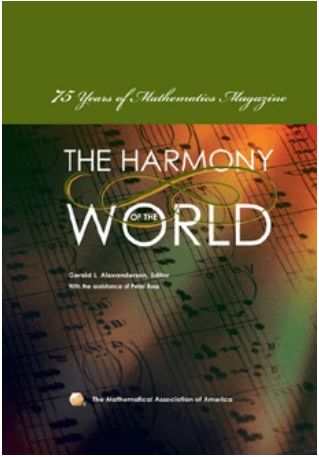
Marian University

MathPath First Plenary/July 19, 2018

Outline

Outline

The Harmony of the World



A Number System with an Irrational Base

A NUMBER SYSTEM WITH AN IRRATIONAL BASE

George Bergman

The reader is probably familiar with the binary system and the decimal system and probably understands the basis for any others of that type, such as the trinary or duodecimal. However, I have developed a system that is based, not on an integer, or even a rational number, but on the irrational number τ (tau), otherwise known as the "golden section", approximately 1.618033989 in value, and equal to $(1+\sqrt{5})/2$.

In order to understand this system, one must comprehend two peculiarities of the number τ. They are based on tau's distincive property [1] that

$$\tau^n = \tau^{n-1} + \tau^{n-2}$$

(a.) Take any approximation (A_1) of τ. Taking the reciprocal, we get a number (a_1) that is proportionately the same distance from $1/\tau$ as A_1 was from τ, but arithmetically nearer. Adding 1,[2] we get a number (A_2) that is proportionately nearer τ than a_1 was to $1/\tau$ but arithmetically just as near. Since a_1 is arithmetically nearer than A_1, A_2 is nearer in both respects to τ than A_1. Repeating the process of taking the reciprocal and adding 1, we approach τ. Now, taking 1 as A_1, and expressing our approximations of τ (i.e. A_1, A_2, A_3, etc.) as fractions, we get

The Cover Letter

Dear Mr. James:
The paper presented is the work of my twelve year old boy who took more than a year to gather the courage to submit it for editorial scrutiny. You may be interested to know that when my son first received a subscription to *Mathematics Magazine* about two and a half years ago, he was aghast to note that he couldn't understand a single thing in it. With each successsive issue, however, his understanding unfolded (he is a self-taught mathematician) until now he awaits each issue with eagerness, and recently was able to submit his solution to one of the Proposals published in your last issue. He considers his subscription to *Mathematics Magazine* one of the finest presents he ever received!
Sincerely yours, Sylvia Bergman

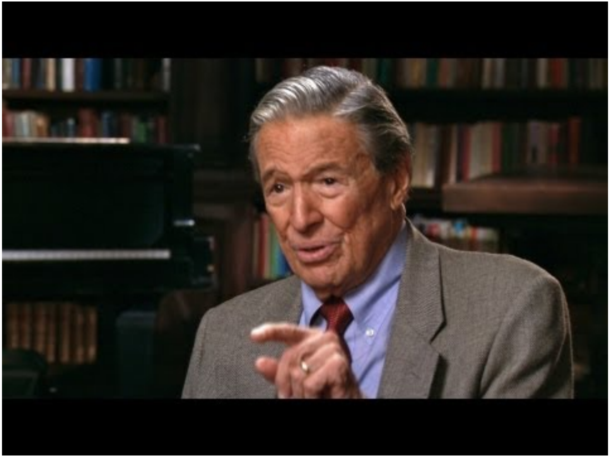
Outline

George Bergman

- Born 22 July 1943
- Attended Stuyvesant High School in New York City
- Got his Ph. D. in 1968 from Harvard under John Tate (my thesis grand-advisor!)
- Was a professor at Cal-Berkeley (now retired)
- Published 2 books and over 100 papers (algebra and mathematical logic)

A Recent Picture

Mike Wallace Interview



Outline

Bergman's Opening

The reader is probably familiar with the binary system and the decimal system and probably understands the basis for any others of that type, such as the trinary or duodecimal. However, I have developed a system that is based, not on an integer, or even a rational number, but on the irrational number τ (tau),....

Definition

- Suppose that the natural number $b > 1$ is the base
- The digits are natural numbers between 0 and $b - 1$
- The number $(a_n a_{n-1} a_{n-2} \dots a_0)_b$ equals $a_n b^n + a_{n-1} b^{n-1} + a_{n-2} b^{n-2} + \cdots + a_0 b^0$
- Using a dot to divide the digits, one can also write fractions
- In general,

$$(a_n a_{n-1} \dots a_1 a_0 . c_1 c_2 c_3 \dots)_b = \sum_{k=0}^{n} a_k b^k + \sum_{k=1}^{\infty} c_k b^{-k}.$$

Base 10

4327

means

$$4 \cdot 10^3 + 3 \cdot 10^2 + 2 \cdot 10^1 + 7 \cdot 10^0$$

Binary

$$101100101_2 = ?_{10}$$

Binary

$$101100101_2$$

means

$$1\cdot 2^8+0\cdot 2^7+1\cdot 2^6+1\cdot 2^5+0\cdot 2^4+0\cdot 2^3+1\cdot 2^2+0\cdot 2^1+1\cdot 2^0 = 357_{10}$$

Hexadecimal

$$165_{16} = ?_{10}$$

Hexadecimal

$$165_{16}$$

means

$$1 \cdot 16^2 + 6 \cdot 16^1 + 5 \cdot 16^0 = 357_{10}$$

Base 10 to Binary

$$122_{10} = ?_2$$

Base 10 to Binary

	1	R 1
2	3	R 1
2	7	R 1
2	15	R 0
2	30	R 1
2	61	R 0
2	122	

The binary representation of 122_{10} is 1111010_2.

Base 10 to Hexadecimal

$$170128_{10} = ?_{16}$$

Base 10 to Hexadecimal

$$
\begin{array}{rll}
 & \underline{2} & R\ 9 \\
16 | & \underline{41} & R\ 8 \\
16 | & \underline{664} & R\ 9 \\
16 | & \underline{10633} & R\ 0 \\
16 | & 170128 &
\end{array}
$$

The hexadecimal representation of 170128_{10} is 29890_{16}.

Outline

Definition

- Let τ equal the positive solution to $x^2 - x - 1 = 0$.
- Thus, $\tau = (1 + \sqrt{5})/2$.
- Approximating, $\tau \approx 1.61803$.
- Note that $\tau^2 = \tau + 1$.

AKA

$$\tau = \phi$$

AKA “The Golden Ratio”

Distinctive Property

- "In order to understand this system, one must comprehend two peculiarities of the number τ."
- They are based on tau's distinctive property that

$$\tau^n = \tau^{n-1} + \tau^{n-2}.$$

Peculiarity One

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \cdots \longrightarrow ?$$

$$1, 2, 1.5, 1.666\ldots, 1.6, 1.625, 1.615382\ldots, \cdots \longrightarrow ?$$

Peculiarity One

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \cdots \longrightarrow \tau$$

In other words, If f_n is the n-th term in the Fibonacci Sequence, setting $f_1 = 1, f_2 = 1$, then

$$\lim_{n \to \infty} (f_{n+1}/f_n) = \tau$$

Peculiarity One

Theorem

$\lim_{n\to\infty}(f_{n+1}/f_n) = \tau$

Proof.

1. The defining property of the squence is $f_{n+1} = f_n + f_{n-1}$.
2. Using this, $f_{n+1}/f_n = (f_n + f_{n-1})/f_n$
3. Simplifying, $f_{n+1}/f_n = 1 + \frac{1}{f_n/f_{n-1}}$
4. Let $x = \lim_{n\to\infty}(f_{n+1}/f_n)$. Then, $x = 1 + 1/x$.
5. I.e., $x^2 = x + 1$. Since x is positive, $x = \tau$. □

Peculiarity One

Theorem

$\lim_{n\to\infty}(f_{n+1}/f_n) = \tau$

Proof.

1. The defining property of the squence is $f_{n+1} = f_n + f_{n-1}$.
2. Using this, $f_{n+1}/f_n = (f_n + f_{n-1})/f_n$
3. Simplifying, $f_{n+1}/f_n = 1 + \frac{1}{f_n/f_{n-1}}$
4. Let $x = \lim_{n\to\infty}(f_{n+1}/f_n)$. Then, $x = 1 + 1/x$.
5. I.e., $x^2 = x + 1$. Since x is positive, $x = \tau$. □

Peculiarity One

Theorem

$\lim_{n\to\infty}(f_{n+1}/f_n) = \tau$

Proof.

1. The defining property of the squence is $f_{n+1} = f_n + f_{n-1}$.
2. Using this, $f_{n+1}/f_n = (f_n + f_{n-1})/f_n$
3. Simplifying, $f_{n+1}/f_n = 1 + \frac{1}{f_n/f_{n-1}}$
4. Let $x = \lim_{n\to\infty}(f_{n+1}/f_n)$. Then, $x = 1 + 1/x$.
5. I.e., $x^2 = x + 1$. Since x is positive, $x = \tau$. □

Peculiarity One

Theorem

$\lim_{n\to\infty}(f_{n+1}/f_n) = \tau$

Proof.

1. The defining property of the squence is $f_{n+1} = f_n + f_{n-1}$.
2. Using this, $f_{n+1}/f_n = (f_n + f_{n-1})/f_n$
3. Simplifying, $f_{n+1}/f_n = 1 + \frac{1}{f_n/f_{n-1}}$
4. Let $x = \lim_{n\to\infty}(f_{n+1}/f_n)$. Then, $x = 1 + 1/x$.
5. I.e., $x^2 = x + 1$. Since x is positive, $x = \tau$.

Peculiarity One

Theorem

$\lim_{n\to\infty}(f_{n+1}/f_n) = \tau$

Proof.

1. The defining property of the squence is $f_{n+1} = f_n + f_{n-1}$.
2. Using this, $f_{n+1}/f_n = (f_n + f_{n-1})/f_n$
3. Simplifying, $f_{n+1}/f_n = 1 + \frac{1}{f_n/f_{n-1}}$
4. Let $x = \lim_{n\to\infty}(f_{n+1}/f_n)$. Then, $x = 1 + 1/x$.
5. I.e., $x^2 = x + 1$. Since x is positive, $x = \tau$. □

Peculiarity Two

Any integral power of τ can be expressed in the form

$$\tau^n = A\tau + B,$$

where A and B are numbers in the Fibonacci Sequence.

Peculiarity Two

- $\tau^1 = 1\tau + 0$
- $\tau^2 = 1\tau + 1$
- $\tau^3 = \tau^2 + \tau^1 = (1\tau + 1) + (1\tau + 0) = 2\tau + 1$
- $\tau^4 = \tau^3 + \tau^2 = (2\tau + 1) + (1\tau + 1) = 3\tau + 2$
- $\tau^5 = \tau^4 + \tau^3 = (3\tau + 2) + (2\tau + 1) = 5\tau + 3$

Peculiarity Two

- $\tau^1 = 1\tau + 0$
- $\tau^2 = 1\tau + 1$
- $\tau^3 = \tau^2 + \tau^1 = (1\tau + 1) + (1\tau + 0) = 2\tau + 1$
- $\tau^4 = \tau^3 + \tau^2 = (2\tau + 1) + (1\tau + 1) = 3\tau + 2$
- $\tau^5 = \tau^4 + \tau^3 = (3\tau + 2) + (2\tau + 1) = 5\tau + 3$

Peculiarity Two

- $\tau^1 = 1\tau + 0$
- $\tau^2 = 1\tau + 1$
- $\tau^3 = \tau^2 + \tau^1 = (1\tau + 1) + (1\tau + 0) = 2\tau + 1$
- $\tau^4 = \tau^3 + \tau^2 = (2\tau + 1) + (1\tau + 1) = 3\tau + 2$
- $\tau^5 = \tau^4 + \tau^3 = (3\tau + 2) + (2\tau + 1) = 5\tau + 3$

Peculiarity Two

- $\tau^1 = 1\tau + 0$
- $\tau^2 = 1\tau + 1$
- $\tau^3 = \tau^2 + \tau^1 = (1\tau + 1) + (1\tau + 0) = 2\tau + 1$
- $\tau^4 = \tau^3 + \tau^2 = (2\tau + 1) + (1\tau + 1) = 3\tau + 2$
- $\tau^5 = \tau^4 + \tau^3 = (3\tau + 2) + (2\tau + 1) = 5\tau + 3$

Peculiarity Two

- $\tau^1 = 1\tau + 0$
- $\tau^2 = 1\tau + 1$
- $\tau^3 = \tau^2 + \tau^1 = (1\tau + 1) + (1\tau + 0) = 2\tau + 1$
- $\tau^4 = \tau^3 + \tau^2 = (2\tau + 1) + (1\tau + 1) = 3\tau + 2$
- $\tau^5 = \tau^4 + \tau^3 = (3\tau + 2) + (2\tau + 1) = 5\tau + 3$

Peculiarity Two

Theorem

For any positive integer n, $\tau^n = f_n\tau + f_{n-1}$.

Proof.

1. We know, $\tau^n = \tau^{n-1} + \tau^{n-2}$.
2. By induction, we have $\tau^n = (f_{n-1}\tau + f_{n-2}) + (f_{n-2}\tau + f_{n-3})$.
3. Rearraning, $\tau^n = (f_{n-1} + f_{n-2})\tau + (f_{n-2} + f_{n-3})$.
4. By the defining property of the sequence, $\tau^n = f_n\tau + f_{n-1}$. □

Peculiarity Two

Theorem

For any positive integer n, $\tau^n = f_n\tau + f_{n-1}$.

Proof.

1. We know, $\tau^n = \tau^{n-1} + \tau^{n-2}$.
2. By induction, we have $\tau^n = (f_{n-1}\tau + f_{n-2}) + (f_{n-2}\tau + f_{n-3})$.
3. Rearraning, $\tau^n = (f_{n-1} + f_{n-2})\tau + (f_{n-2} + f_{n-3})$.
4. By the defining property of the sequence, $\tau^n = f_n\tau + f_{n-1}$.

Peculiarity Two

Theorem

For any positive integer n, $\tau^n = f_n\tau + f_{n-1}$.

Proof.

1. We know, $\tau^n = \tau^{n-1} + \tau^{n-2}$.
2. By induction, we have $\tau^n = (f_{n-1}\tau + f_{n-2}) + (f_{n-2}\tau + f_{n-3})$.
3. Rearraning, $\tau^n = (f_{n-1} + f_{n-2})\tau + (f_{n-2} + f_{n-3})$.
4. By the defining property of the sequence, $\tau^n = f_n\tau + f_{n-1}$.

□

Peculiarity Two

Theorem

For any positive integer n, $\tau^n = f_n\tau + f_{n-1}$.

Proof.

1. We know, $\tau^n = \tau^{n-1} + \tau^{n-2}$.
2. By induction, we have $\tau^n = (f_{n-1}\tau + f_{n-2}) + (f_{n-2}\tau + f_{n-3})$.
3. Rearraning, $\tau^n = (f_{n-1} + f_{n-2})\tau + (f_{n-2} + f_{n-3})$.
4. By the defining property of the sequence, $\tau^n = f_n\tau + f_{n-1}$. $\square$

Negative Powers of τ

"Can this be applied to negative powers of τ? We don't know any Fibonacci numbers before 1, but it is easy to find them."

Fibonacci Numbers Before 1

$$\ldots, -21, 13, -8, 5, -3, 2, -1, 1, 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots$$

Fibonacci Numbers Before 1

Theorem

For any positive integer y, $f_{-y} = (-1)^{y+1} f_y$.

Proof.

- Sketch....
- Use the defining property of the Fibonacci Sequence.
- Prove by induction.

$\square$

List of Powers of τ

$$
\begin{aligned}
\tau^{-5} &= 5\tau - 8 \\
\tau^{-4} &= -3\tau + 5 \\
\tau^{-3} &= 2\tau - 3 \\
\tau^{-2} &= -1\tau + 2 \\
\tau^{-1} &= 1\tau - 1
\end{aligned}
\qquad
\begin{aligned}
\tau^{0} &= 0\tau + 1 \\
\tau^{1} &= 1\tau + 0 \\
\tau^{2} &= 1\tau + 1 \\
\tau^{3} &= 2\tau + 1 \\
\tau^{4} &= 3\tau + 2 \\
\tau^{5} &= 5\tau + 3
\end{aligned}
$$

Outline

Symbols and Rule

- Like the binary system, the only symbols are 0 and 1
- Unlike the binary system, it has the rule $100 = 011$
- This is a general rule; place the decimal point anywhere.
- This is a restatement of $\tau^n = \tau^{n-1} + \tau^{n-2}$.

Symbols and Rule

- Like the binary system, the only symbols are 0 and 1
- Unlike the binary system, it has the rule $100 = 011$
- This is a general rule; place the decimal point anywhere.
- This is a restatement of $\tau^n = \tau^{n-1} + \tau^{n-2}$.

Symbols and Rule

- Like the binary system, the only symbols are 0 and 1
- Unlike the binary system, it has the rule $100 = 011$
- This is a general rule; place the decimal point anywhere.
- This is a restatement of $\tau^n = \tau^{n-1} + \tau^{n-2}$.

Trivial Examples

- Obviously, $0_{10} = 0_\tau$.
- Also, $1 = \tau^0$, so $1_{10} = 1_\tau$.

Trivial Examples

- Obviously, $0_{10} = 0_\tau$.
- Also, $1 = \tau^0$, so $1_{10} = 1_\tau$.

Other Examples

We know,

$$\tau^1 = 1\tau + 0 \quad \text{and} \quad \tau^{-2} = -1\tau + 2.$$

Therefore,

$$\tau^1 + \tau^{-2} = (1\tau + 0) + (-1\tau + 2) = 2.$$

In other words,

$$2_{10} = 10.01_\tau.$$

Other Examples

We know,

$$\tau^2 = 1\tau + 1 \quad \text{and} \quad \tau^{-2} = -1\tau + 2.$$

Therefore,

$$\tau^2 + \tau^{-2} = (1\tau + 1) + (-1\tau + 2) = 3.$$

In other words,

$$3_{10} = 100.01_\tau.$$

Other Examples

We know,

$$\tau^0 = 1 \quad \text{and} \quad \tau^2 + \tau^{-2} = 3.$$

Therefore,

$$\tau^2 + \tau^{-2} + \tau^0 = 4.$$

In other words,

$$4_{10} = 101.01_\tau.$$

Outline

Nonuniqueness of Representations

$$\begin{aligned} 2_{10} &= 10.01_\tau \\ &= 1.11_\tau \\ &= 10.0011_\tau \\ &= 10.001011_\tau \\ &= 1.101011_\tau \end{aligned}$$

Simplest Form

- The form in which there are no two 1s in succession.
- Therefore, cannot be acted upon by **simplification** $(011 = 100)$.
- To convert a number to its simplest form, repeatedly simplify the leftmost pair of consecutive 1s.

Simplification Example

$$\begin{aligned} 100101.111001 &= 100110.011001 \\ &= 101000.011001 \\ &= 101000.100001 \end{aligned}$$

A τ-representation is Always Possible

Theorem

For every positive integer n, there is a corresponding finite sequence of distinct integers $k_1, k_2, \ldots, k_m$ such that $n = \tau^{k_1} + \tau^{k_2} + \cdots + \tau^{k_m}$.

- Bergman implicitly makes this claim, and demonstrates a method that should always work.
- C. Rousseau (1995) published a follow-up article in *Mathematics Magazine* ("The Phi Number System Revisited") in which he proves the theorem (and a little bit more) using some algebraic number theory.

Getting from n to $n + 1$

- Put n in its simplest form.
- Convert n into the form in which there is a 0 in the units column.
- Add a 1 to the units column to produce $n + 1$.
- Put $n + 1$ in its simplest form.

Example

$$\begin{aligned} 4_{10} &= 101.01_{\tau} \\ &= 101.0011_{\tau} \\ &= 100.1111_{\tau} \end{aligned}$$

Now add 1 to get 5.

$$\begin{aligned} 5_{10} &= (100.1111 + 1)_{\tau} \\ &= 101.1111_{\tau} \\ &= 110.0111_{\tau} \\ &= 1000.0111_{\tau} \\ &= 1000.1001_{\tau} \end{aligned}$$

Method of Conversion

- If there is 0 in the units column, you are finished.
- If there is 1, look in the τ^{-2} column. If there is 0 there, expand the 1 and you are done.
- If there is 1, look in the τ^{-4} column. If there is 0 there, expand the 1 in the τ^{-2} column and the 1 in the units column and you are done.
- If there is 1, look in the τ^{-6} column. If there is 0 there, expand the 1 in the τ^{-4} column, the 1 in the τ^{-2} column and the 1 in the units column and you are done.
- Etc.

Method of Conversion

- If there is 0 in the units column, you are finished.
- If there is 1, look in the τ^{-2} column. If there is 0 there, expand the 1 and you are done.
- If there is 1, look in the τ^{-4} column. If there is 0 there, expand the 1 in the τ^{-2} column and the 1 in the units column and you are done.
- If there is 1, look in the τ^{-6} column. If there is 0 there, expand the 1 in the τ^{-4} column, the 1 in the τ^{-2} column and the 1 in the units column and you are done.
- Etc.

Method of Conversion

- If there is 0 in the units column, you are finished.
- If there is 1, look in the τ^{-2} column. If there is 0 there, expand the 1 and you are done.
- If there is 1, look in the τ^{-4} column. If there is 0 there, expand the 1 in the τ^{-2} column and the 1 in the units column and you are done.
- If there is 1, look in the τ^{-6} column. If there is 0 there, expand the 1 in the τ^{-4} column, the 1 in the τ^{-2} column and the 1 in the units column and you are done.
- Etc.

Method of Conversion

- If there is 0 in the units column, you are finished.
- If there is 1, look in the τ^{-2} column. If there is 0 there, expand the 1 and you are done.
- If there is 1, look in the τ^{-4} column. If there is 0 there, expand the 1 in the τ^{-2} column and the 1 in the units column and you are done.
- If there is 1, look in the τ^{-6} column. If there is 0 there, expand the 1 in the τ^{-4} column, the 1 in the τ^{-2} column and the 1 in the units column and you are done.
- Etc.

Method of Conversion

- If there is 0 in the units column, you are finished.
- If there is 1, look in the τ^{-2} column. If there is 0 there, expand the 1 and you are done.
- If there is 1, look in the τ^{-4} column. If there is 0 there, expand the 1 in the τ^{-2} column and the 1 in the units column and you are done.
- If there is 1, look in the τ^{-6} column. If there is 0 there, expand the 1 in the τ^{-4} column, the 1 in the τ^{-2} column and the 1 in the units column and you are done.
- Etc.

What if?

The method fails for $1.01010101\ldots$. How can this be simplified?

Endless Fractions

The endless fraction $1.01010101\ldots$ equals 10.

$$\begin{aligned}
1+\tau^{-2}+\tau^{-4}+\tau^{-6}+\cdots &= \frac{1}{1-\tau^{-2}} \\
&= \frac{\tau^2}{\tau^2-1} \\
&= \frac{\tau+1}{\tau} \\
&= 1+\frac{1}{\tau} \\
&= \tau
\end{aligned}$$

Exercise

Show that in the Tau System,

$$1 = 0.10101010\ldots.$$

(Just as in the decimal system,

$$1 = 0.999999\ldots.)$$

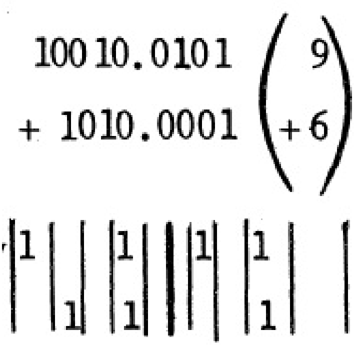
Several Examples

0 – 0	5 – 1000.1001	10 – 10100.0101
1 – 1	6 – 1010.0001	11 – 10101.0101
2 – 10.01	7 – 10000.0001	12 – 100000.101001
3 – 100.01	8 – 10001.0001	13 – 100010.001001
4 – 101.01	9 – 10010.0101	14 – 100100.110110

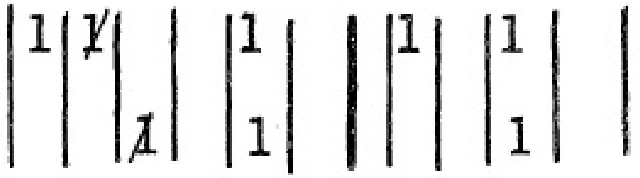
Examples of the basic processes

Outline

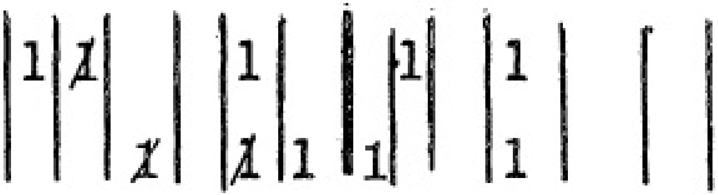
Addition



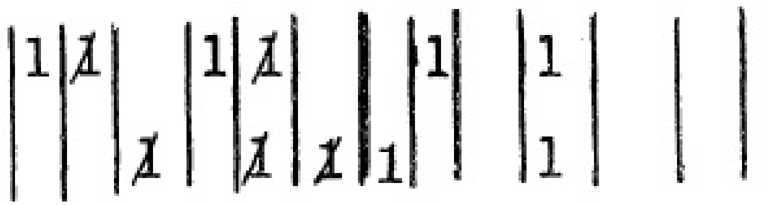
Addition



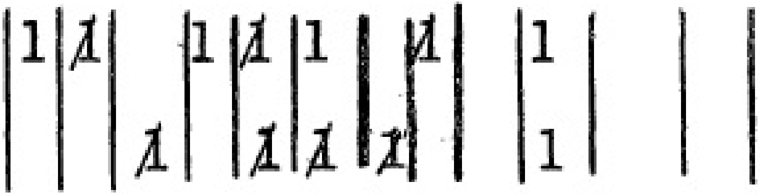
Addition



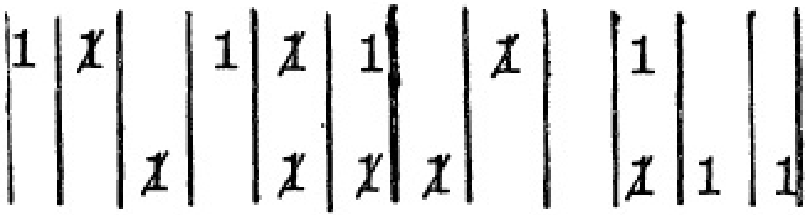
Addition



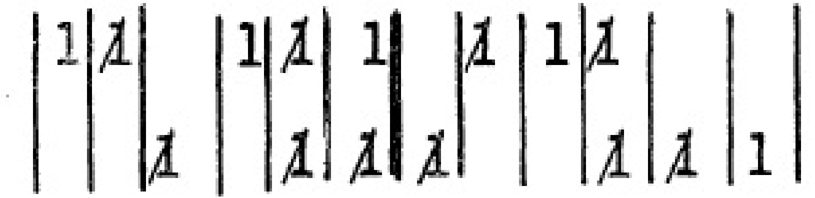
Addition



Addition



Addition



Addition

$$10010.0101 + 1010.0001 = 100101.001001$$

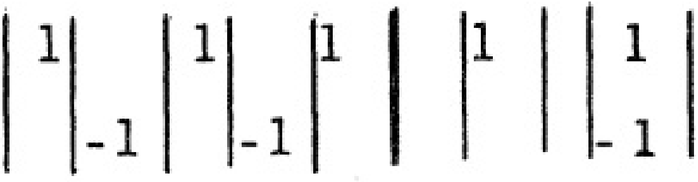
$$9 + 6 = 15$$

Subtraction

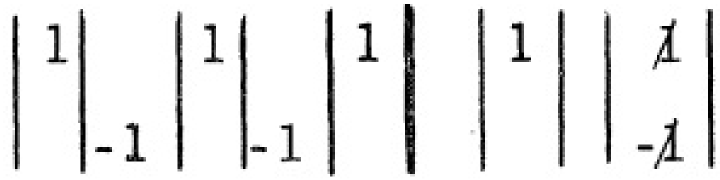
$$10010.0101 - 1010.0001 = ?$$

$$11 - 6 = 5$$

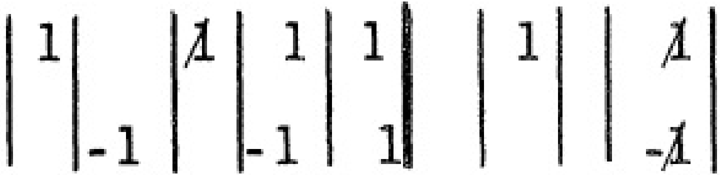
Subtraction



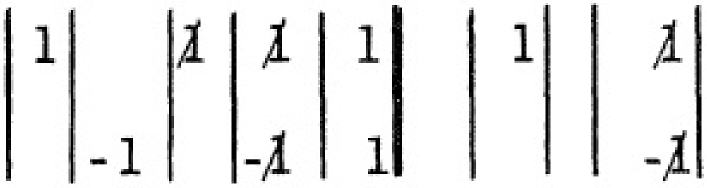
Subtraction



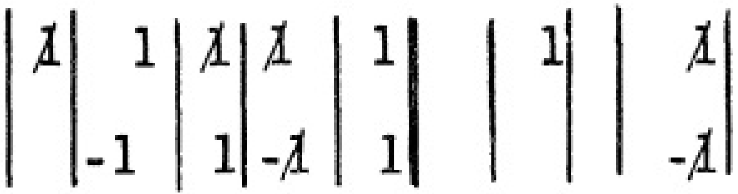
Subtraction



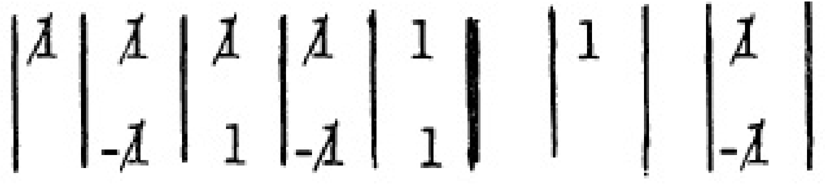
Subtraction



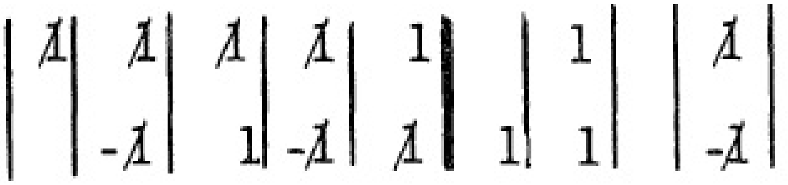
Subtraction



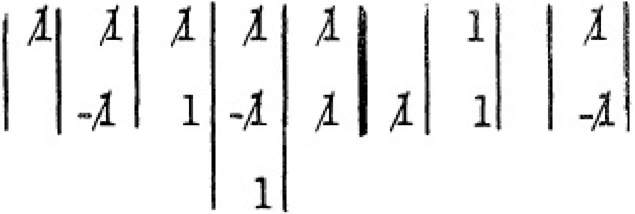
Subtraction



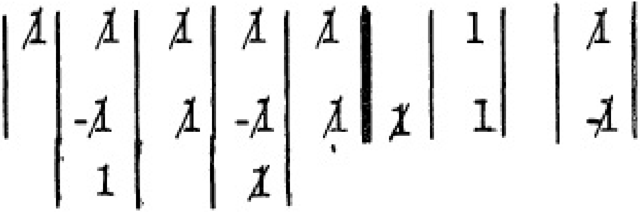
Subtraction



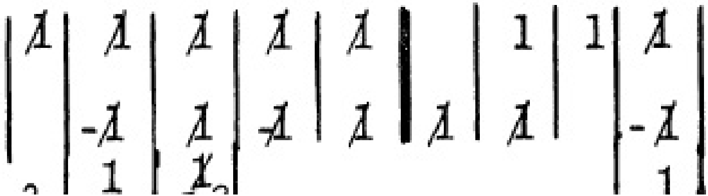
Subtraction



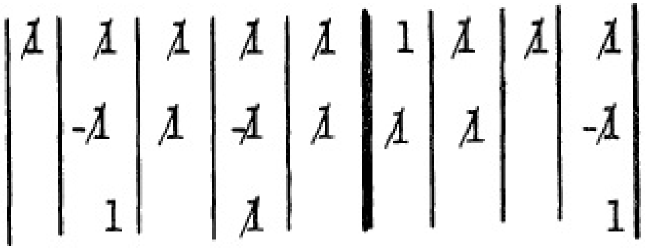
Subtraction



Subtraction



Subtraction



Subtraction

$$10010.0101 - 1010.0001 = 1000.1001$$

Multiplication

"Multiplication involves nothing new. We simply place the partial products as we do in the decimal system, and add."

Multiplication

$$\begin{array}{r} 101.01 \\ \times\, 100.01 \\ \hline 100000.101001 \end{array}$$

Division

"Division is quite different in this system, and is, in fact, rather odd. Thie only things it has in common with ordinary division are the basic principles behind it, the way the example looks, and the movement of the 'decimal point' to eliminate any figures to the right of it in the divisor."

Division

$$100000.101001 \div 10.01 = 1010.0001$$

Outline

Fractions

$$1/2 = .\underline{010}010010010\ldots\ldots\ldots\ldots$$

$$1/3 = .\underline{00101000}0010100000101000\ldots\ldots\ldots$$

$$1/4 = .\underline{001000}001000001000001000\ldots\ldots\ldots\ldots$$

$$1/5 = .\underline{00010010101001001000}001001010100100100\ldots\ldots\ldots$$

$$1/10 = .\underline{0000100001000101000010100010101010001001010000001001000100}$$
$$\underline{000}00001\ldots$$

Fractions

Exercise: Prove that no fractions can be terminating in this system.

Reflections

- It's never too early to start making your contribution to mathematics.
- You don't have to win a Fields Medal to have a successful mathematical career.
- Be thankful for those who have encouraged and enabled your progress in mathematics.
- Do your part to encourage others to love the beauty of mathematics.

References

G. Alexanderson and P. Ross, Eds.
The Harmony of the World: 75 Years of Mathematics Magazine.
Mathematical Association of America, 2007.

G. Bergman.
A Number System with an Irrational Base.
Mathematics Magazine, 31(1):198–110, 1957.

C. Rousseau.
The Phi Number System Revisited.
Mathematics Magazine, 68(4):283–284, 1995.